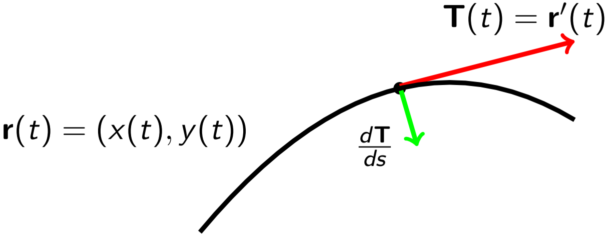
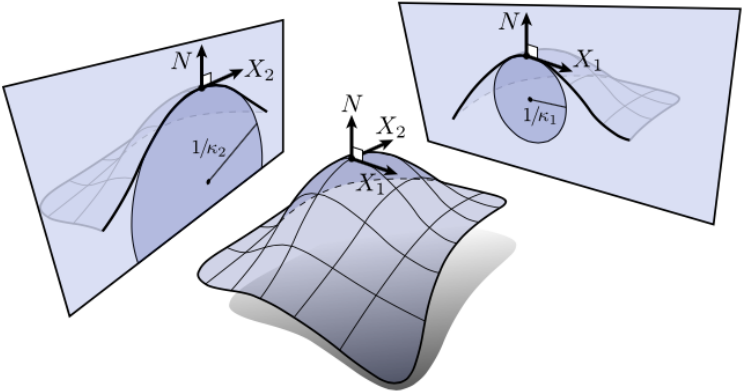
Two beautiful Minds



$$\text{Krumning:} \quad \kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{|x'y'' - y'x''|}{((x')^2 + (y')^2)^{\frac{3}{2}}}$$

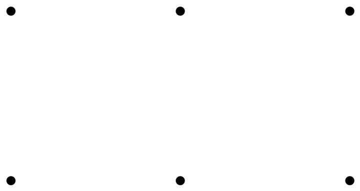


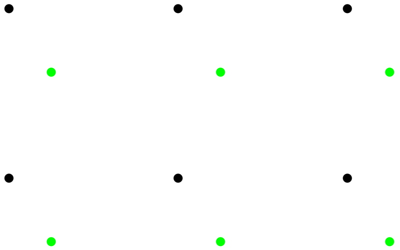
N
X_2
$1/\kappa_2$
N
X_2
X_1
N
X_1
$1/\kappa_1$

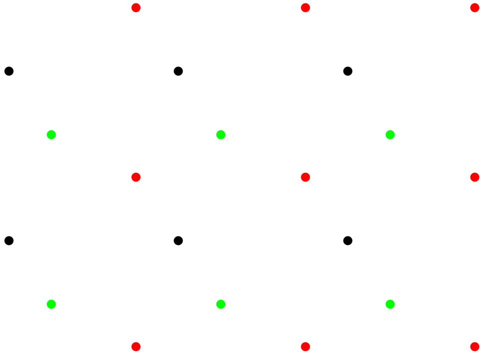
Carl Friedrich Gauss (1777-1855)

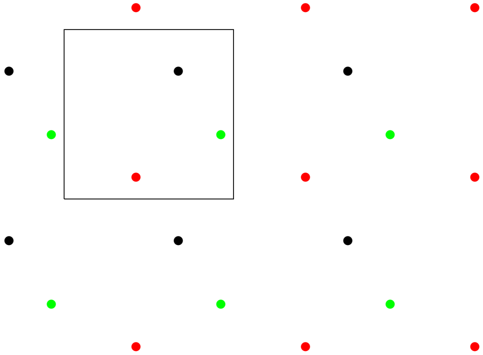
Theorema Egregium (1827):
Si superficies curva in quamcunque aliam superficiem explicatur, mensura curvaturae in singulis punctis invariata manet.

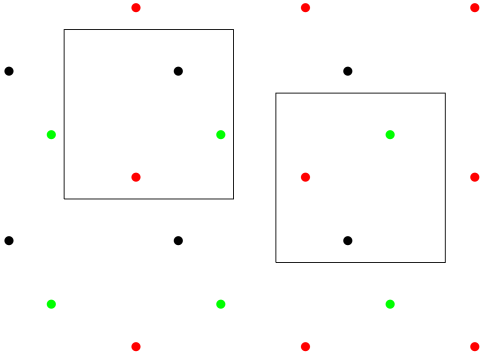
Dersom en buet flate er utviklet på en vilkårlig annen flate, så vil krumningen i hvert punkt forbli uendret.

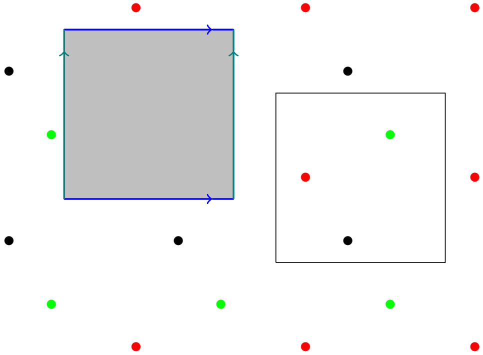








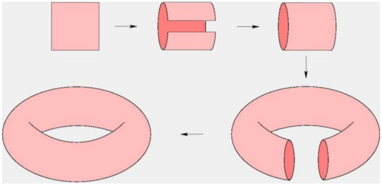




Georg Friedrich Bernhard Riemann (1826-1866)

Über die Hypothesen welche der Geometrie zu Grunde liegen (1854):

Bekanntlich setzt die Geometrie sowohl den Begriff des Raumes, als die ersten Grundbegriffe für die Constructionen in Raume als etwas Gegebenes voraus. Sie giebt von ihnen nur Nominaldefinitionen, während die wesentlichen Bestimmungen in Form von Axiomen auftreten. Das Verhältniss dieser Voraussetzungen bleibt dabei in Dunkeln; man sieht weder ein, ob und in wie weit ihre Verbindung nothwendig, noch a priori, ob sie möglich ist.



Nash-Kuiper-teoremet. La (M, g) være en Riemannsk mangfoldighet av dimensjon m og $f : M \to \mathbb{R}^n$ en kort C^∞-embedding inn i et Euklidsk rom $\mathbb{R}^n$, hvor $n \geq m + 1$. Da finnes for vilkårlig $\epsilon > 0$ en embedding $f_\epsilon : M \to \mathbb{R}^n$ av type C^1, slik at

(i) f_ϵ er en isometri, dvs. for alle par av tangentvektorer $v, w \in T_x(M)$ så vil

$$g(v, w) = \langle df_\epsilon(v), df_\epsilon(w) \rangle$$

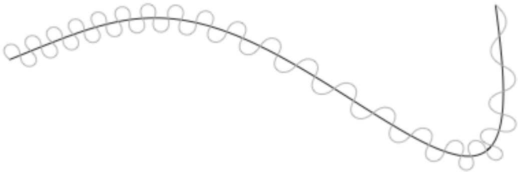
(ii) f_ϵ har punktvis avstand til f mindre enn ϵ:

$$|f(x) - f_\epsilon(x)| < \epsilon, \;\; \forall x \in M.$$

C: Plan regulær kurve, parametrisert ved $\mathbf{r} : [0,1] \to \mathbb{R}^2$.
Kurven gjennomløpes med hastighet $v_0(t) = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\|$.
$v(t)$: En annen hastighetsfunksjon, som oppfyller $v(t) \geq v_0(t)$ for alle $t \in [0,1]$.

Finn ny kurve C', parametrisert ved $\mathbf{r}' : [0,1] \to \mathbb{R}^2$ med gjennomløpshastighet v.

Problem: Gjør dette slik at kravet $\|\mathbf{r}'(t) - \mathbf{r}(t)\| \leq \epsilon$ er oppfyllt for vilkårlig valgt $\epsilon > 0$.

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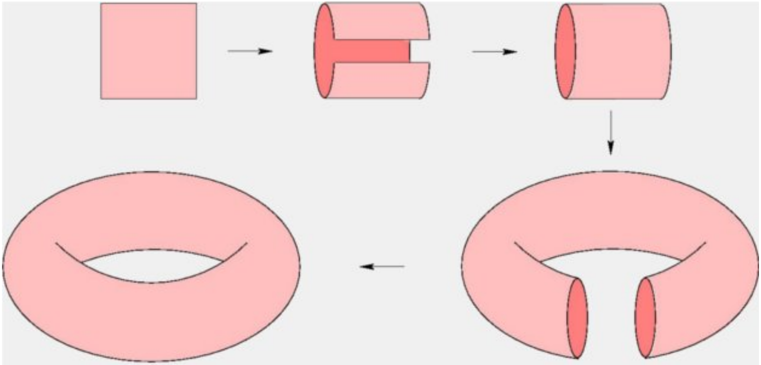
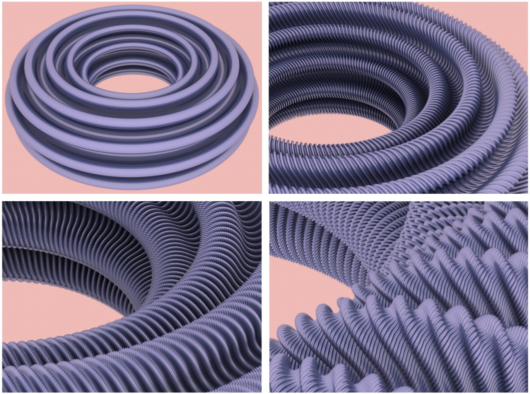




Illustrasjon av en isometrisk embedding av en flat torus inn i $\mathbb{R}^3$.

(Kilde: HEVEA Project/PNAS)

COMMUNICATIONS ON PURE AND APPLIED MATHEMATICS, VOL. VI, 337–394 (1953)

The Weyl and Minkowski Problems in Differential Geometry in the Large

By LOUIS NIRENBERG

Introduction

The problems of Weyl and Minkowski treated in this paper are two classical embedding problems of differential geometry in the large. Such problems usually reduce to questions concerning nonlinear differential equations and those treated here lead to nonlinear equations of elliptic character. Consequently, much of the paper is concerned with questions in the field of elliptic differential equations.

The first problem, which was considered by H. Weyl [29] in 1916, is the problem of the realization by a convex surface in Euclidean 3-space of a differential geometric metric of positive curvature given on the unit sphere. In other words, one is given a positive definite quadratic form defined at every point of the unit sphere—which in local parameters (u, v) takes the form

$$ds^2 = E(u, v)\, du^2 + 2F(u, v)\, du\, dv + G(u, v)\, dv^2,$$

with ds^2 invariant under parameter change, and such that the Gauss curvature of the form is everywhere *positive*. Does there exist a closed convex surface which may be mapped one-to-one onto the sphere so that its first fundamental form, in terms of parameters on the sphere, is ds^2?

The quadratic form defines the Riemann metric of an abstract Riemannian manifold homeomorphic to the sphere, and the problem may be formulated as: can this manifold be embedded into Euclidean 3-space?

A proof of the possibility of such an embedding is given here, under the assumption that the coefficients of the quadratic form ds^2 possess derivatives up to the fourth order.

In addition a solution of the Minkowski problem (formulated in [20], see also [9] chapter 13) is presented. This problem is the following: Given a positive function $K(\vec{n})$ defined on the unit sphere (here $\vec{n}$ represents the inner unit normal to the sphere), does there exist a closed convex surface having $K(\vec{n})$ as its Gauss curvature at the point on the surface where the inner normal is $\vec{n}$? The function $K(\vec{n})$ is assumed to satisfy the condition, which holds for any regular closed convex surface,

$$\int K(\vec{n})\vec{n}\, d\omega\, (\vec{n}) = 0,$$

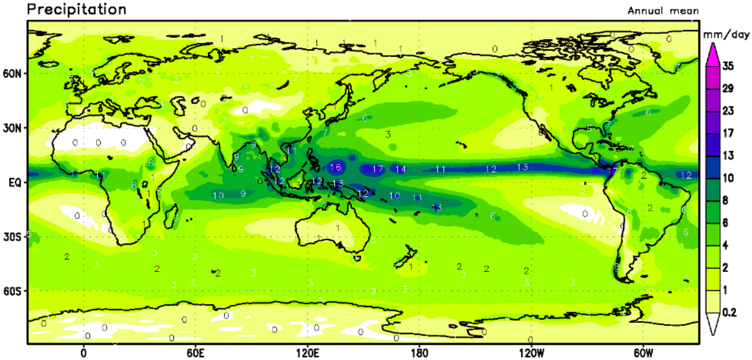
Hermann Minkowski (1864-1909)

Minkowski-problemet (1903).

Gitt en strengt positivt, reell funksjon f definert på kuleflaten S^2, finn en strengt konveks, kompakt flate $\Sigma \subset \mathbb{R}^3$ slik at Gauss-krumningen $\kappa(x)$ til Σ i punktet x er presis lik $f(\mathbf{n}(x))$, hvor $\mathbf{n}(x)$ er normalen til Σ i x.

Hermann Weyl (1885-1955)

Weyl-problemet (1916). Betrakt kuleflaten S^2 utstyrt med en Riemannsk metrikk g slik at den tilhørende Gauss-krumningen er positiv overalt. Finnes det da en global isometrisk C^2-embedding $X : (S^2, g) \to (\mathbb{R}^3, \sigma)$, hvor σ er standard-metrikken på $\mathbb{R}^3$?


Precipitation
Annual mean
mm/day
35
29
23
17
13
10
8
6
4
2
1
0.2
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W

Ulik størrelse på kroppsdelene reflekterer tettheten av nerveceller.

(Kilde: Natural History Museum, London)

"Partielle differensialligninger brukes for å beskrive grunnleggende lover for fenomener innen fysikk, kjemi, biologi og andre vitenskaper. De er også nyttige i analysen av geometriske objekter, slik en rekke vellykkede eksempler fra de siste tiårene viser.

John Nash og Louis Nirenberg har spilt en ledende rolle i utviklingen av denne teorien, gjennom løsning av fundamentale problemer og introduksjon av dype ideer. Deres gjennombrudd har utviklet seg til anvendelige og robuste teknikker, som nå er sentrale redskaper for studiet av ikke-lineære partielle differensialligninger."

(Kilde: Abelkomiteens begrunnelse)

Jean-Baptiste Joseph Fourier (1768-1830)

Théorie analytique de la chaleur (1822).

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$

Funksjonen $u = u(t, \mathbf{x})$ måler temperaturen i et gitt punkt på en bestemt tid og likningen gir den matematiske modellen for varmetransport. Likningen uttrykker matematisk det faktum at i et punkt hvor temperaturen er lavere enn i omgivelsene, så vil temperaturen stige over tid.

Jean le Rond d'Alembert (1717-1783)

Leonhard Euler (1707-1783)

Bølgelikningen (1746-56).

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0$$

Funksjonen $u = u(t, \mathbf{x})$ beskriver amplituden til bølgen, slik den utvikler seg i tid og rom.

Elliptisk PDE:

$$\nabla^2 u = f$$

- Ingen tidskoordinat i likningen
- Løsningene er fullstendig romlige
- Elliptiske likninger modellerer statiske fysiske eller geomtriske problemer

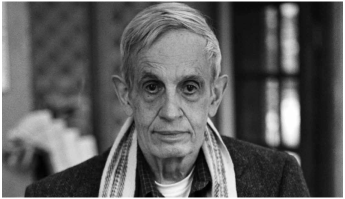
F.eks. isometriske embeddingsproblemer: La

$$\mathbf{u} = \left(u^1(x,y), u^2(x,y), u^3(x,y)\right)$$

beskrive en embedding $\mathbb{R}^2 \to \mathbb{R}^3$, der $g = (g_{ij})$ er metrikken på $\mathbb{R}^2$ og $\mathbb{R}^3$ har standard Euklidsk metrikk. En isometrisk embedding er gitt av diff.likningssystemet

$$\frac{\partial u^1}{\partial x_i}\frac{\partial u^1}{\partial x_j} + \frac{\partial u^1}{\partial x_i}\frac{\partial u^1}{\partial x_j} + \frac{\partial u^1}{\partial x_i}\frac{\partial u^1}{\partial x_j} = g_{ij}$$

for $1 \leq i, j \leq 2$.



Regularity issues are a daily concern in the study of partial differential equations, sometimes for the sake of rigorous proofs, and sometimes for the precious qualitative insights that they provide about the solutions.

It was a breakthrough in the field when Nash proved, in parallel with De Giorgi, the first Hölder estimates for solutions of linear elliptic equations in general dimensions without any regularity assumption on the coefficients; among other consequences, this provided a solution to Hilbert's 19th problem about the analyticity of minimizers of analytic elliptic integral functionals.

(Kilde: Abelkomiteens begrunnelse)

A few years after Nash's proof, Nirenberg established, together with Agmon and Douglis, several innovative regularity estimates for solutions of linear elliptic equations with L^p data, which extend the classical Schauder theory and are extremely useful in applications where such integrability conditions on the data are available. These works founded the modern theory of regularity, which has since grown immensely, with applications in analysis, geometry and probability, even in very rough, non-smooth situations.

(Kilde: Abelkomiteens begrunnelse)

18. mai 2015 kl 11:00 *Kunnskapsministeren overrekker Holmboeprisen*, Aulaen, Oslo katedralskole

18. mai 2015 kl 17:00 *Kransenedlegging ved Abelmonumentet*, Slottsparken, Oslo

19. mai 2015 kl 14:00 *Abelprisutdeling i Universitetets Aula*, Universitetets Aula, Oslo

20. mai 2015 kl 10:00 *Abelforelesningene 2015*, Georg Sverdrups Hus, Universitetet i Oslo

ABELFORELESNINGENE

ONSDAG 20. MAI 2015

Georg Sverdrups hus, Universitetet i Oslo
Kl. 10:00 - 15:00

Forelesningene er åpne, men lunsjen krever påmelding.
Send e-post til abelprisen@dnva.no
Se mer informasjon om forelesningene og Abelprisen 2015 på www.abelprisen.no

ABEL PRISEN

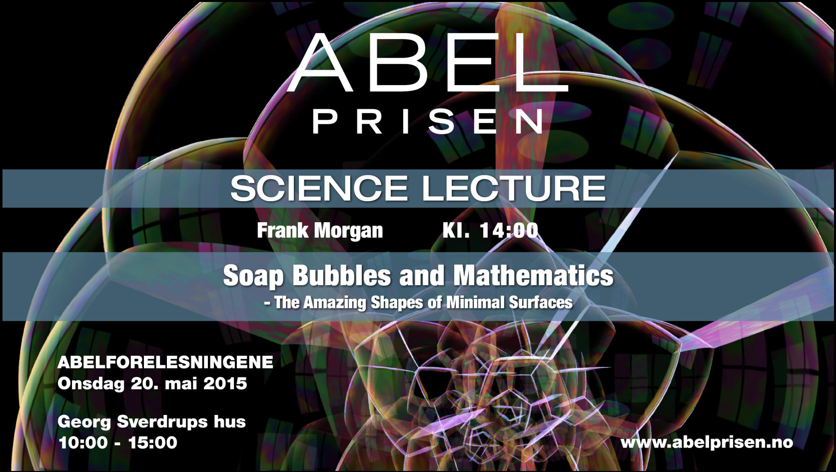
DET NORSKE VIDENSKAPS-AKADEMI

kl. 10:00  **Introductions by John F. Nash, Jr. and Louis Nirenberg**

kl. 10:30  **Camillo De Lellis:**
Surely you're joking, Mr. Nash?

kl. 12:00  **Tristan Rivière:**
Exploring the unknown, the work of Louis Nirenberg on Partial Differential Equations

kl. 14:00  Science Lecture with **Frank Morgan:**
Soap Bubbles and Mathematics
- The Amazing Shapes of Minimal Surfaces


ABEL
PRISEN
SCIENCE LECTURE
Frank Morgan
Kl. 14:00
Soap Bubbles and Mathematics
- The Amazing Shapes of Minimal Surfaces
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Onsdag 20. mai 2015
Georg Sverdrups hus
10:00 - 15:00
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